Chapter 4

Dipoles and Monopoles

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4-1 INTRODUCTION

Since the publication of the first edition of this handbook several sections in the original chapter on linear antennas have become outdated and have been deleted in this edition. The availability of computer programs[1] for finding the impedance and other characteristics of antennas, particularly linear antennas, makes parametric tabulation of limited usage. Only some essential formulas and design data are therefore included in this chapter.

For the entire subject of linear antennas, the book by R. W. P. King[2] remains authoritative. Another book[3] by the same author on the tables of antenna characteristics contains the most comprehensive data on the characteristics of cylindrical antennas. Calculations on circular-loop antennas and some simple arrays are also found there.

A section on the effective height of antennas is included in this chapter. The usage of this parameter in describing the transmitting and receiving characteristics of linear antennas and other simple structures is discussed in detail. In addition, material on the general formulation of receiving antennas is included so that engineers can apply the formulation for design purposes or for estimation of the coupling effect between elements made up of both linear and other types of antennas.

Antennas in lossy media are of great current interest. Unfortunately, the subject cannot be covered in this chapter because of limited space. The book by King and Smith[4] on antennas in matter can be consulted for this subject, particularly for linear antennas embedded in a lossy medium.

4-2 CYLINDRICAL DIPOLES

Impedance as a Function of Length and Diameter

The impedance characteristics of cylindrical antennas have been investigated by many writers. Theoretical work has mainly been confined to relatively thin antennas (length-to-diameter ratio greater than 15), and the effect of the junction connecting the antenna proper and the transmission line is usually not considered. Among various theories, the induced-emf method[5] of computing the impedance of a cylindrical antenna based upon a sinusoidal distribution is still found to be very useful. The formula derived from this method is extremely simple. It is, however, valid only when the half length of a center-driven antenna is not much longer than a quarter wavelength. In practice, this is the most useful range. To eliminate unnecessary computations, the formula has been reduced to the following form:[6]

$$Z_i = R(k\ell) - j\left[120\left(\ln\frac{\ell}{a} - 1\right)\cot k\ell - X(k\ell)\right] \qquad \textbf{(4-1)}$$

where Z_i = input impedance, Ω, of a center-driven cylindrical antenna of total length 2ℓ and of radius a

$k\ell = 2\pi(\ell/\lambda)$ = electrical length, corresponding to ℓ, measured in radians

The functions $R(k\ell)$ and $X(k\ell)$ are tabulated in Table 4-1 and plotted in Fig. 4-1 for the range $k\ell \leq \pi/2$. For calculation purposes, these two functions can be approximated to within 0.5 Ω by the following simple third-order polynomials:

$$R(k\ell) = -0.4787 + 7.3246k\ell + 0.3963(k\ell)^2 + 15.6131(k\ell)^3$$

$$X(k\ell) = -0.4456 + 17.0082k\ell - 8.6793(k\ell)^2 + 9.6031(k\ell)^3$$

TABLE 4-1 Functions $R(k\ell)$ and $X(k\ell)$ Contained in the Formula of the Input Impedance of a Center-Driven Cylindrical Antenna

$k\ell$	$R(k\ell)$	$X(k\ell)$	$k\ell$	$R(k\ell)$	$X(k\ell)$
0	0	0	0.9	18.16	15.01
0.1	0.1506	1.010	1.0	23.07	17.59
0.2	0.7980	2.302	1.1	28.83	20.54
0.3	1.821	3.818	1.2	35.60	23.93
0.4	3.264	5.584	1.3	43.55	27.88
0.5	5.171	7.141	1.4	52.92	32.20
0.6	7.563	8.829	1.5	64.01	38.00
0.7	10.48	10.68	$\pi/2$	73.12	42.46
0.8	13.99	12.73			

When the length of the antenna is short compared with a wavelength but still large compared with its radius, the same formula reduces to

$$(Z_i)_{\text{short}} = 20(k\ell)^2 - j120(k\ell)^{-1}\left(\ln\frac{\ell}{a} - 1\right) \tag{4-2}$$

For antennas of half length greater than a quarter wavelength, a number of refined theories provide formulas for the computation of the impedance function. None of them, however, is simple enough to be included here. As far as numerical computation is concerned, Schelkunoff's method[7] is relatively simpler than Hallén's.[2] It should be emphasized that all these theories are formulated by using an idealized model in which the terminal condition is not considered.

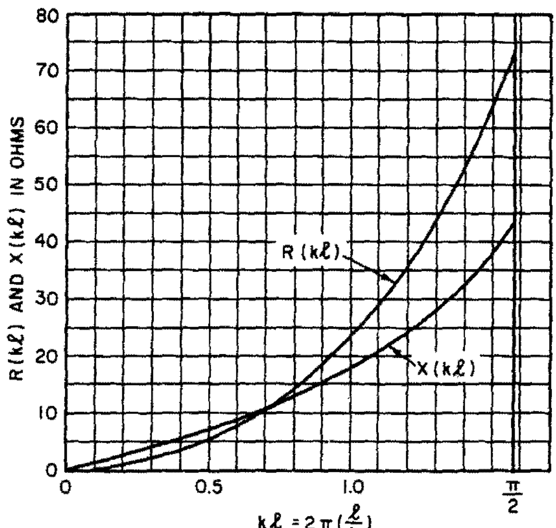

FIG. 4-1 The functions $R(k\ell)$ and $X(k\ell)$.

In practice, the antenna is always fed by a transmission line. The complete system may have the appearances shown in Fig. 4-2. The effective terminal impedance of the line (often referred to as the antenna impedance) then depends not only upon the length and the diameter of the antenna but also upon the terminal condition. In cases *a* and *b*, the impedance would also be a function of the size of the ground plane. For a given terminal condition the variation of the impedance of a cylindrical antenna as a function of the length and the diameter of the antenna is best shown in the experimental work of Brown and Woodward.[8] The data cover a wide range of values of the length-to-diameter ratio. Two useful sets of curves are reproduced in Figs. 4-3 and 4-4. The impedance refers to a cylindrical antenna driven by a coaxial line through a large circular ground plane placed on the surface of the earth. The arrangement is similar to the one sketched in Fig. 4-2*a*. The length and diameter of the antenna are measured in degrees; i.e., a length of one wavelength is equivalent to 360°. If the effects due to the terminal condition and finite-size ground plane are neglected, the impedance would correspond to one-half of the impedance of a center-driven antenna (Fig. 4-2*c*). In using these data for design purposes, one must take into consideration the actual terminal condition as compared with the condition specified by these two authors. In particular, the maximum value of the resistance and the resonant length of the antenna may change considerably if the base capacitance is excessive.

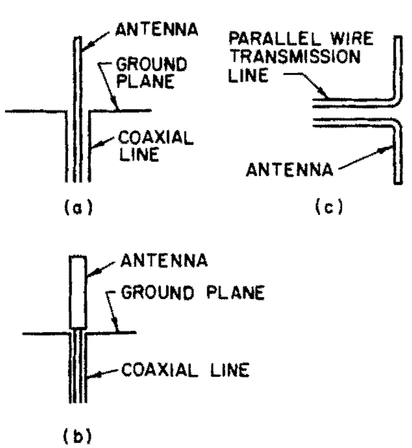

FIG. 4-2 Driving an antenna by transmission lines.

Effect of Terminal Conditions

Many authors have attempted to determine the equivalent-circuit elements corresponding to different terminal conditions. Schelkunoff and Friis[9] have introduced the concepts of *base capacitance* and *near-base capacitance* to explain the shift of the impedance curve as the terminal condition is changed. Similar interpretations have been given by King[10] for a cylindrical antenna driven by a two-wire line or by a coaxial line and by Whinnery[11] for a biconical antenna driven by a coaxial line. The importance of the terminal condition in effecting the input impedance of the antenna is shown in Figs. 4-5 and 4-6. They are again reproduced from Brown and Woodward's paper. Because of the large variation of the effective terminal impedance of the line with changes in the geometry of the terminal junction, one must be cautious when using the theoretical results based upon isolated antennas. For junctions possessing simple geometry, the static method of Schelkunoff and Friis, King, and Whinnery can be applied to estimate the shunt capacitance of the junction. The latter then can be combined with the impedance of the antenna proper to evaluate the resultant impedance. For intricate junctions, accurate information can be obtained only by direct measurement.

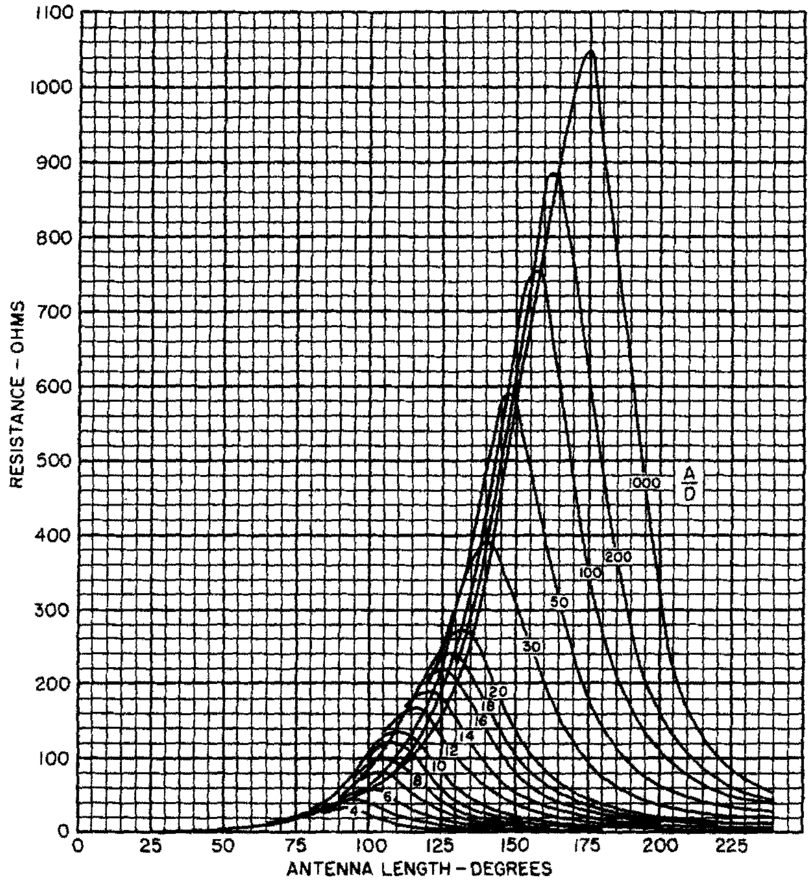

FIG. 4-3 Antenna resistance versus antenna length A when a constant ratio of length to diameter A/D is maintained. Here the length and diameter are held constant while the frequency is changed.

Equivalent Radius of Noncircular Cross Sections

As far as the impedance characteristics and radiation pattern are concerned, a thin cylindrical antenna with a noncircular cross section behaves like a circular cylindrical antenna with an equivalent radius. In stating this characteristic, the terminal effect is, of course, not considered. The equivalent radius of many simply shaped cross sections can be found by the method of conformal mapping.[12] For an elliptical cross section the following simple relation exists:

$$a_{eq} = \tfrac{1}{2}(a + b) \qquad (4\text{-}3)$$

where a = major axis of ellipse
b = minor axis of ellipse

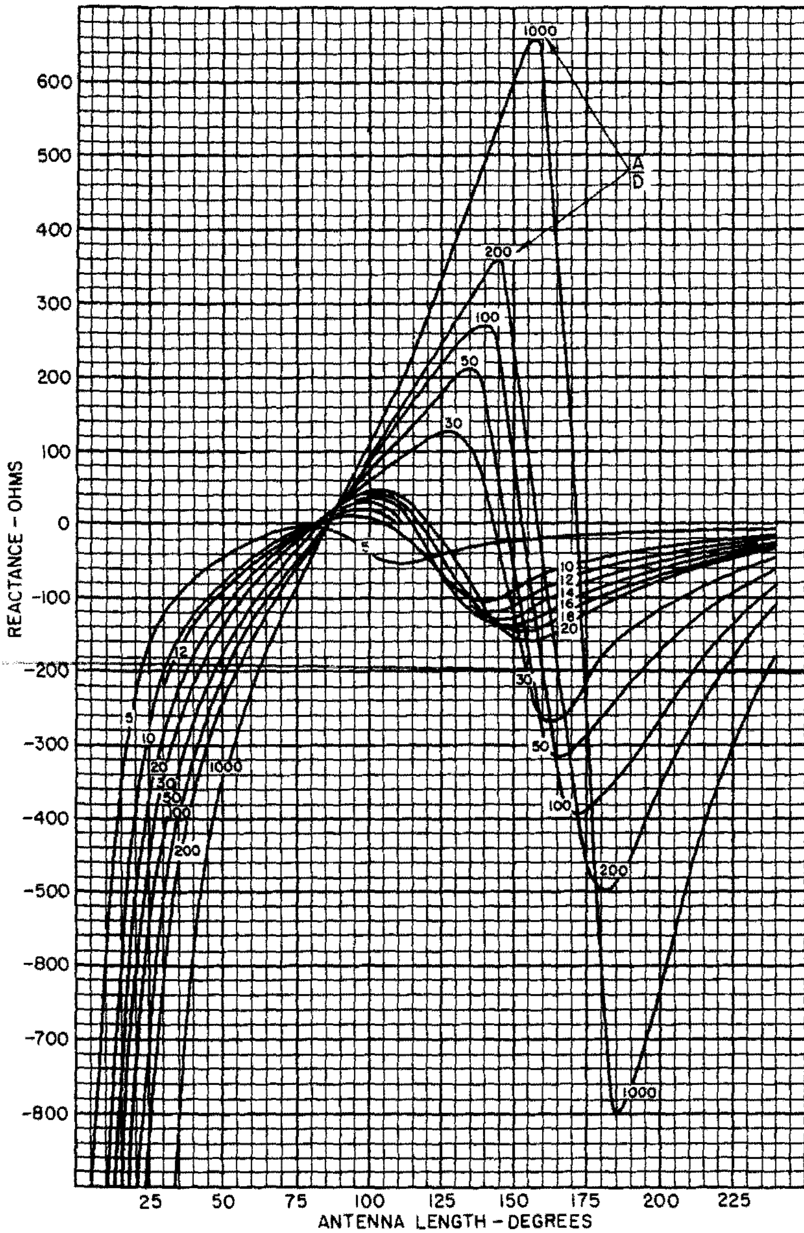

FIG. 4-4 Reactance curves corresponding to the resistance curves of Fig. 4-3.

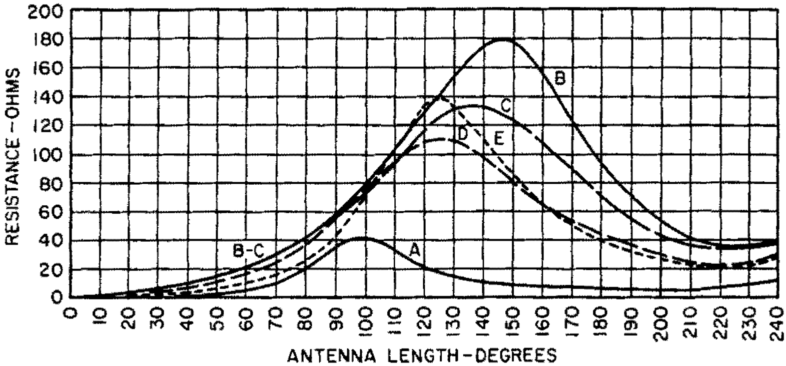

FIG. 4-5 Resistance as a function of antenna length *A*. The diameter *D* is 20.6°. Curve *A*: The arrangement shown in Fig. 4-2*b*. Curve *B*: The arrangement of Fig. 4-2*a* with the diameter of the outer conductor equal to 74°. The characteristic impedance of the transmission line is 77.0 Ω. Curve *C*: The outer-conductor diameter is 49.5°, and the transmission line has a characteristic impedance of 52.5 Ω. Curve *D*: The diameter of the outer conductor is 33°. The characteristic impedance is 28.3 Ω. Curve *E*: This curve was obtained by tuning out the base reactance with an inductive reactance of 65.0 Ω.

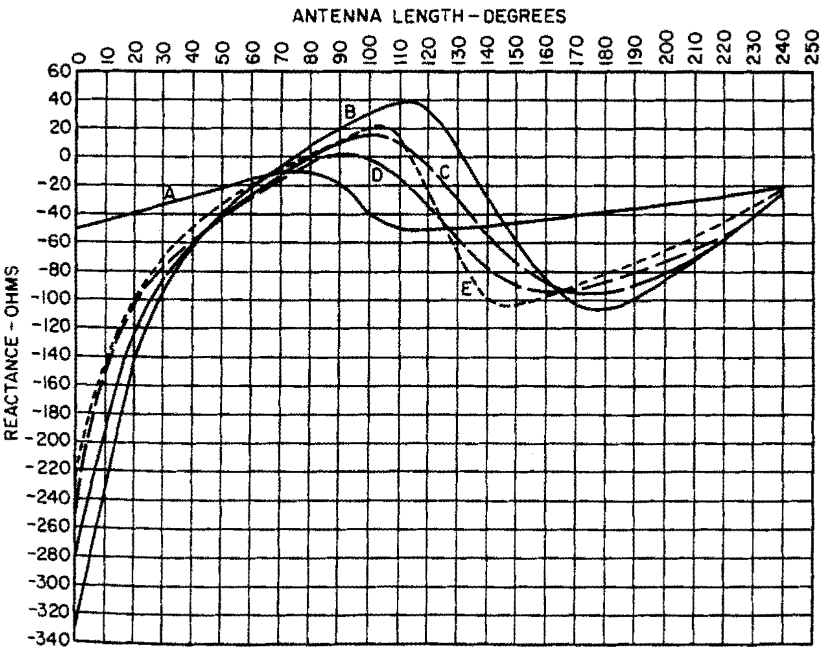

FIG. 4-6 Reactance curves corresponding to the resistance curves of Fig. 4-5.

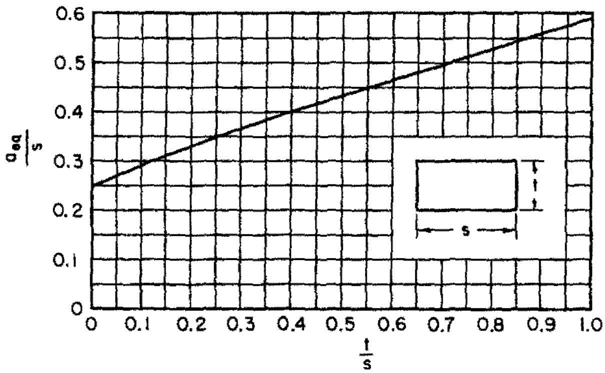

FIG. 4-7 Equivalent radius a_{eq} of a rectangle as a function of the ratio of thickness t to width s.

For a rectangular cross section the result is plotted in Fig. 4-7. In the case of a strip, Eq. (4-3) and Fig. 4-7 give the identical result. When the cross section has the form of a regular polygon, the result is tabulated in Table 4-2. The equivalent radius of two parallel cylinders of radius ρ_1 and ρ_2 separated by a distance d between the centers is given by[13]

$$\ln \rho_e = \frac{1}{(\rho_1 + \rho_2)^2} (\rho_1^2 \ln \rho_1 + \rho_2^2 \ln \rho_2 + 2\rho_1\rho_2 \ln d) \qquad \textbf{(4-4)}$$

Formulas for the equivalent radius of three cylinders and an angle strip are found in Ref. 13.

Patterns as a Function of Length and Diameter

In this subsection only the radiation pattern of center-driven cylindrical antennas is discussed. For base-driven antennas, the patterns depend very much upon the size of the ground plane. The subject will be discussed in Sec. 4-8.

The radiation pattern of a center-driven cylindrical antenna in general depends upon its length and thickness. The terminal condition which plays an important role in determining its impedance has a negligible effect on the pattern. For thin antennas, the calculated pattern obtained by assuming a sinusoidal current distribution is a good

TABLE 4-2 Equivalent Radius of a Regular Polygon

n	3	4	5	6
a_{eq}/a	0.4214	0.5903	0.7563	0.9200

n = number of sides.

a = radius of the outscribed circle.

approximation of the actual pattern. Thus, with an assumed current distribution of the form

$$I(z) = I_0 \sin k(\ell - |z|) \qquad +\ell \geq z \geq -\ell \tag{4-5}$$

the radiation field, expressed in a spherical coordinate system, is given by

$$E_\theta = \frac{j\eta I_0 e^{jkR}}{2\pi R}\left[\frac{\cos(k\ell\cos\theta) - \cos k\ell}{\sin\theta}\right] \tag{4-6}$$

where $\eta = (\mu/\epsilon)^{1/2} = 120\pi \quad \Omega$

θ = angle measured from axis of dipole, or z axis

The field pattern is obtained by evaluating the *magnitude* of the term contained in the brackets of Eq. (4-6). Some of the commonly referred-to patterns are sketched in Fig. 4-8. Comparing those patterns with the actual patterns of a thin cylindrical antenna

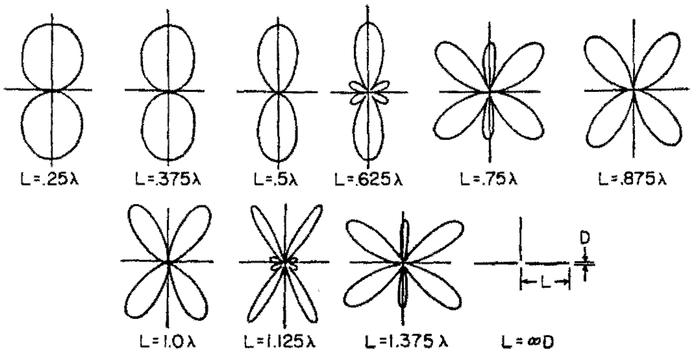

FIG. 4-8 Radiation patterns of center-driven dipoles if sinusoidal current distribution is assumed.

obtained by measurement, one finds that the theoretical patterns based upon a sinusoidal current distribution do not contain the following information:

1 The nulls between the lobes, except the *natural null* in the direction of the axis, are actually not vanishing.

2 The phase of the field varies continuously from lobe to lobe instead of having a sudden jump of 180° between the adjacent lobes.

3 The actual patterns vary slightly with respect to the diameter of the antenna instead of being independent of the thickness.

Depending upon the particular applications, some of the fine details may require special attention. In most cases, the idealized patterns based upon a sinusoidal current distribution give us sufficient information for design purposes.

When the half length ℓ of the antennas is less than about one-tenth wavelength, Eq. (4-6) is well approximated by

$$E_\theta = \frac{j\eta I_0 (k\ell)^2 e^{jkR}}{4\pi R}\sin\theta \tag{4-7}$$

The figure-eight pattern resulting from the plot of the sine function is a characteristic not only of short cylindrical antennas but also of all small dipole-type antennas. Equations (4-6) and (4-7) are also commonly used to evaluate the directivity of linear antennas. The directivity is defined as

$$D = \frac{\text{maximum radiation intensity}}{\text{average radiation intensity}} \tag{4-8}$$

For a short dipole, D is equal to 1.5. The directivity of a half-wave dipole ($\ell = \lambda/4$) is equal to 1.64. The half-wave dipole is often used as a reference antenna to describe the gain of more directive antennas, particularly arrays made of dipoles.

4-3 BICONICAL DIPOLES

Impedance as a Function of Length and Cone Angle

When the angles of a symmetrical biconical antenna (Fig. 4-9) are small, the input impedance of the antenna can be calculated by using Schelkunoff's formula.[7] Some

FIG. 4-9 A biconical dipole.

sample curves are shown in Fig. 4-10. While the biconical antenna is an excellent theoretical model for studying the essential property of a dipole-type antenna, small-angle biconical antennas are seldom used in practice. Wide-angle biconical antennas or their derived types such as discones, however, are frequently used as broadband antennas. The broadband impedance characteristics occur when the angle of the cones, θ_0 of Fig. 4-9, lies between 30 and 60°. The exact value of θ_0 is not critical. Usually it is chosen so that the characteristic impedance of the biconical dipole matches as closely as possible the characteristic impedance of the line which feeds the antenna. The characteristic impedance of a biconical dipole as a function of the angle is plotted in Fig. 4-11. For a conical monopole driven against an infinitely large ground plane, the characteristic impedance and the input impedance of the antenna are equal to half of the corresponding values of a dipole. Several formulas[15] are available for computing the input impedance of wide-angle biconical antennas. Actual computation has been confined to a very few specific values of θ_0.[16,17] More complete information is available from the experimental data obtained by Brown and Woodward.[18] Two curves are reproduced in Figs. 4-12 and 4-13. The case corresponding to $\alpha = 0°$ represents a cylindrical antenna having a diameter of 2.5 electrical degrees at a frequency of 500 MHz, since the feed point was kept fixed at that diameter.

Patterns of the Biconical Dipole

The radiation patterns of biconical dipoles have been investigated theoretically by Papas and King.[19] Figure 4-14 shows the patterns of a 60°-flare-angle ($\theta_0 = 30°$) conical dipole for various values of ka, where $k = 2\pi/\lambda$ and a = half length of the